ACCURATE SOLUTIONS OF THE ONE-PHASE PROBLEM ON THE MELTING OF A SOLID WEDGE

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UDC 536.42

In quasisteady formulation accurate solutions are obtained for the problem of melting of a semiinfinite solid wedge in which its dihedral angles have an aperture angle πt, where t is a simple fraction.

The present work is a continuation of [1, 2] and uses the same formulation of the problem on the melting of solids. In Cartesian coordinates, the quasisteady heat conduction for a solid, melting along the z-axis, has the form [1]:

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} + \frac{v_0}{a}\frac{\partial U}{\partial z} = 0 \tag{1}$$

with the boundary conditions

$$U|_\Sigma = 0;\ U|_{h\in\Omega} \leqslant 0;\ U|_{h\to\infty} \to U_\infty, \tag{2}$$

where U(x, y, z) is the temperature at the point of a solid with coordinates x, y, z; Ω is the region of a melted solid confined by a surface performing forward movement with a velocity v_0 along the z-axis; h is the shortest distance between the point (x, y, z) and the solid boundary.

Introduce dimensionless variables:

$$\xi_i = (x\cos\varphi_i + y\cos\psi_i + z\cos\theta_i)\, v_0/a,\quad i = 1,\ 2,\ 3,$$

where $\cos\varphi_i$, $\cos\psi_i$, $\cos\theta_i$ are the direction cosines of the normal to the plane, the equation of which is $\xi_i = 0$. The geometric term ξ_i is the distance from the point with coordinates x, y, z to the plane $\xi_i = 0$ multiplied by v_0/a.

Consider the problem on the melting of the solid wedge, the faces of which are the planes $\xi_i = 0$.

The heat-conduction equation (1) is now written in new variables

$$\frac{\partial^2 U}{\partial \xi_1^2} + \frac{\partial^2 U}{\partial \xi_2^2} + \frac{\partial^2 U}{\partial \xi_3^2} + 2B\frac{\partial^2 U}{\partial \xi_1 \partial \xi_2} + 2C\frac{\partial^2 U}{\partial \xi_1 \partial \xi_3} + 2D\frac{\partial^2 U}{\partial \xi_2 \partial \xi_3} + A_1\frac{\partial U}{\partial \xi_1} + A_2\frac{\partial U}{\partial \xi_2} + A_3\frac{\partial U}{\partial \xi_3} = 0, \tag{3}$$

where $B = \cos V_{12}$; $C = \cos V_{13}$; $D = \cos V_{23}$; $A_i = \cos\theta_i$; V_{12} is the angle between the normals to the plane $\xi_1 = 0$, $\xi_2 = 0$; V_{13} is the angle between $\xi_1 = 0$, $\xi_3 = 0$, and V_{23} the angle between $\xi_2 = 0$, $\xi_3 = 0$.

The fundamental solution of Eq. (3) is sought in the form

$$U = \exp - (\alpha\xi_1 + \beta\xi_2 + \gamma\xi_3). \tag{4}$$

Substituting Eq. (4) into Eq. (3) for determination of the multipliers α, β, and γ yields the characteristic equation

$$Э(\alpha,\ \beta,\ \gamma) = \alpha^2 + \beta^2 + \gamma^2 + 2B\alpha\beta + 2C\alpha\gamma + 2D\beta\gamma - A_1\alpha - A_2\beta - A_3\gamma = 0. \tag{5}$$

The exponential function (4) is a solution of Eq. (3) if the point (α, β, γ) lies on the ellipsoid (5). The algorithm for determining the spectra of points, allowing a solution for the dihedral angle to be written [2], makes it possible to calculate them.

Voronesh Technological Institute, Russia. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 63, No. 3, pp. 368-374, September, 1992. Original article submitted July 27, 1991.

Let $\gamma = 0$ in the ellipsoid (5). This corresponds to the problem of searching for the spectra $\{\alpha_n^B\}$, $\{\beta_n^B\}$ entering the solution for the dihedral angle formed by the planes $\xi_1 = 0, \xi_2 = 0$. By the known value of the angle between them, the fraction is determined, with the aid of which we write the angle between the normals to the planes $\xi_1 = 0, \xi_2 = 0$; $V_{12} = \pi q/(n + 1)$, $n = 1, 2, ...$; $q = 1, 2, ..., n$. The number n determines the quantity of points of the characteristic ellipse $\mathcal{E}(\alpha, \beta, 0)$ required to write a solution. The spectra $\{\alpha_n^B\}$, $\{\beta_n^B\}$ are found from the recurrence relations (12) from [2]:

$$\begin{gathered}\alpha_n^B = A_1 P_n(B) - A_2 Q_n(B);\ \beta_n^B = -A_1 Q_n(B) + A_2 P_n(B);\\ P_n(B) = 1 - P_{n-2}(B) + 2BQ_{n-1}(B);\ Q_n(B) = -Q_{n-2}(B) + 2BP_{n-1}(B);\\ \alpha_0^B = \beta_0^B = 0;\ B = \cos\frac{\pi q}{n+1};\ P_0 = 0;\ P_1 = 1;\ Q_0 = Q_1 = 0.\end{gathered} \tag{6}$$

The superscript B of the quantities α_n^B and β_n^B denotes that they are calculated from B.

According to (26) from [2], a solution of the two-dimensional problem is written in the form of a closing sum multiplied by U_∞. Write this sum as

$$R_0(\alpha_0, \beta_0) = 1 + \sum_{p=1}^{n} (-1)^p (\langle \alpha_p \beta_{p-1} \rangle + \langle \alpha_{p-1} \beta_p \rangle) + (-1)^n \langle \alpha_n \beta_n \rangle,$$

where $\langle \alpha_i \beta_j \rangle = \exp -(\alpha_i \xi_i + \beta_j \xi_2)$. In the notation of this sum $R_0(\alpha_0, \beta_0)$, the subscript indicates a number of the section $\gamma = \gamma_0$, while the point (α_0, β_0) on the given section of the ellipsoid stands for the origin of spectra.

Repeat this algorithm for the angle between the planes $\xi_1 = 0, \xi_3 = 0$, setting $\beta = 0$ in (5). Determine the fraction $e/(m + 1)$ to write the angle $V_{13} = \pi e/(m + 1)$ between the normals to $\xi_1 = 0$ and $\xi_3 = 0$ where $\cos V_{13} = C$. Using formulas similar to (6), we determine the desired spectra $\{\alpha_m^C\}$, $\{\gamma_m^C\}$ at $\beta = 0$

$$\begin{gathered}\alpha_m^C = A_1 P_m(C) - A_3 Q_m(C);\ \gamma_m^C = -A_1 Q_m(C) + A_3 P_m(C);\\ P_m(C) = 1 - P_{m-2}(C) + 2CQ_{m-1}(C);\ Q_m(C) = -Q_{m-2}(C) + 2CP_{m-1}(C);\\ \alpha_0^C = \gamma_0^C = 0;\ P_0 = 0;\ P_1 = 1;\ Q_0 = Q_1 = 0.\end{gathered} \tag{7}$$

Analogously, for the angle between $\xi_2 = 0$ and $\xi_3 = 0$ we take $\alpha = 0$ in (5) and determine the fraction $s/(r + 1)$ which allows the angle $V_{23} = \pi s/(r + 1)$ between the normals to these planes to be written as $\cos V_{23} = D$.

The spectra $\{\beta_r^D\}$, $\{\gamma_r^D\}$ are determined by formulas as (6) and (7):

$$\begin{gathered}\beta_r^D = A_2 P_r(D) - A_3 Q_r(D);\ \gamma_r^D = -A_2 Q_r(D) + A_3 P_r(D);\\ P_r(D) = 1 - P_{r-2}(D) + 2DQ_{r-1}(D);\ Q_r(D) = -Q_{r-2}(D) + 2DP_{r-1}(D);\\ \beta_0^D = \gamma_0^D = 0;\ P_0 = 0;\ P_1 = 1;\ Q_0 = Q_1 = 0.\end{gathered} \tag{8}$$

For the subsequent consideration, assume that $B = C = D$, $A_1 = A_2 = A_3$. This corresponds to the equal angles between the faces of a trihedron and the same slope of the faces to the z-axis along which melting occurs. In this case we obtain that spectra (6)-(8) have the equal number n of terms and, at the same time, α_p from (6) is equal to α with the same number p found from (7): $\alpha_p^B = \alpha_p^C = \alpha_p$, this is valid both for β_p and γ_p.

Consider cutting of the ellipsoid (5) by the plane $\gamma = \gamma_1$:

$$\begin{gathered}\mathcal{E}_1(\alpha, \beta) = \alpha^2 + \beta^2 + 2B\alpha\beta - A_{11}(\alpha_0, \beta_0)\alpha - A_{21}(\alpha_0, \beta_0)\beta = 0;\\ A_{11}(\alpha_0, \beta_0) = A_1 - 2C\gamma_1;\ A_{21}(\alpha_0, \beta_0) = A_2 - 2D\gamma_1.\end{gathered}$$

Among the ellipsoid points $\mathcal{E}(\alpha, \beta, \gamma) = 0$ written in (7) and (8), the three points $(\alpha_0, \beta_2^D, \gamma_1^D)$, $(\alpha_0, \beta_0, \gamma_1)$, $(\alpha_2^C, \beta_0, \gamma_1^C)$ lie on the ellipse $\mathcal{E}_1(\alpha, \beta)$. From the equation $\mathcal{E}_1(\alpha, \beta) = 0$, other points are determined which together with those three points make the closed spectra $\{\alpha_n\}_1$, $\{\beta_n\}_1$ with the spectra $\gamma = \gamma_1$. The algorithm is as follows. The point $(\alpha_0, \beta_0, \gamma_1)$ is taken as the origin of the spectra, i.e., $\alpha_{0,1} = \alpha_0$; $\beta_{0,1} = \beta_0$; while $\alpha_{1,1}$ and $\beta_{1,1}$ are determined from the equations $\mathcal{E}_1(\alpha_{0,1}, \beta_{1,1}) = 0$ and $\mathcal{E}_1(\alpha_{1,1}, \beta_{0,1}) = 0$; $\alpha_{1,1} = A_{11}(\alpha_0, \beta_0)$; $\beta_{1,1} = A_{21}(\alpha_0, \beta_0)$. All the ensuing $\alpha_{n,1}$ and $\beta_{n,1}$ are found from the equations $\mathcal{E}_1(\alpha_{n+1,1}, \beta_{n,1}) = 0$ and $\mathcal{E}_1(\alpha_{n,1}, \beta_{n\pm 1,1}) = 0$ by recurrence formulas similar to (6):

$$\begin{gathered}\alpha_{n,1} = A_{11}(\alpha_0, \beta_0) P_n(\beta) - A_{21}(\alpha_0, \beta_0) Q_n(B);\\ \beta_{n,1} = -A_{11}(\alpha_0, \beta_0) Q_n(B) + A_{21}(\alpha_0, \beta_0) P_n(B).\end{gathered} \tag{9}$$

Since the polynomials $P_n(B)$ and $Q_n(B)$ in (9) are the same as in (6) and the closure condition of the spectra $\{\alpha_n\}_1$, $\{\beta_n\}_1$ does not depend on $A_{11}(\alpha_0, \beta_0)$ and $A_{21}(\alpha_0, \beta_0)$, as shown in [2], then the newly calculated spectra (9) are deliberately closed for the given β as the spectra (6) are closed. Besides, the number of terms $\alpha_{n,1}$ and $\beta_{n,1}$, forming the closed spectra (6) and (9), is the same.

Note that all three points, known at the beginning of this calculation on the section $\partial_1(\alpha, \beta) = 0$, have entered the spectra (9).

The calculated spectra $\{\alpha_n\}_1$, $\{\beta_n\}_1$ allow us to write the approximate closing sum:

$$R_1(\alpha_0, \beta_0) = 1 + \sum_{p=1}^{n} (-1)^p (\langle \alpha_{p,1}\beta_{p-1,1} \rangle + \langle \alpha_{p-1,1}\beta_{p,1} \rangle) + (-1)^n \langle \alpha_{n,1}\beta_{n,1} \rangle$$

Next, consider cutting of the ellipsoid (5) by the plane $\gamma = \gamma_2$.

Four points written in spectra (7), (8) lie on the ellipse $\partial_2(\alpha, \beta) = 0$. The spectrum (7) contains the points

$$(\alpha_1^C, \beta_0, \gamma_2^C), (\alpha_2^C, \beta_0, \gamma_2^C), \tag{10}$$

while the spectrum (8) –

$$(\alpha_0, \beta_1^D, \gamma_2^D), (\alpha_0, \beta_2^D, \gamma_2^D). \tag{11}$$

Now obtain separately for each given pair of points the other points of the section $\gamma = \gamma_2$ forming the closing spectra. Take the point $(\alpha_1^C, \beta_0, \gamma_2^C)$ from (10) as the origin and introduce new variables:

$$\alpha' = \alpha - \alpha_1^C; \ \beta' = \beta - \beta_0.$$

The equation for the ellipse $\partial_2(\alpha, \beta) = 0$ in new variables has the form:

$$\alpha'^2 + \beta'^2 + 2B\alpha'\beta' - A_{12}(\alpha_1^C, \beta_0)\alpha' - A_{22}(\alpha_1^C, \beta_0)\beta' = 0;$$
$$A_{12}(\alpha_1^C, \beta_0) = A_1 - 2C\gamma_2^C - 2B\beta_0 - 2\alpha_1^C;$$
$$A_{22}(\alpha_1^C, \beta_0) = A_2 - 2D\gamma_2^C - 2B\alpha_1^C - 2\beta_0.$$

Let $\alpha_0' = \beta_0' = 0$. From the equations $\partial_2(\alpha_0', \beta_1') = 0$ and $\partial_2(\alpha_1', \beta_0') = 0$ we determine $\alpha_1' = A_{12}(\alpha_1^C, \beta_0)$ and $\beta_1' = A_{22}(\alpha_1^C, \beta_0)$, while from the equations $\partial_2(\alpha'_{n \pm 1}, \beta'_n) = 0$ and $\partial_2(\alpha_n', \beta_{n \pm 1}) = 0$ α'_n and β'_n are determined by the recurrence formulas similar to (6):

$$\alpha_n' = A_{12}(\alpha_1^C, \beta_0)P_n(B) - A_{22}(\alpha_1^C, \beta_0)Q_n(B);$$
$$\beta_n' = -A_{12}(\alpha_1^C, \beta_0)Q_n(B) + A_{22}(\alpha_1^C, \beta_0)P_n(B).$$

It is easy to see that the newly calculated spectra $\{\alpha'_n\}$, $\{\beta'_n\}$ are closed for the given B as in the case of the sections $\gamma = \gamma_0$ and $\gamma = \gamma_1$. The closing sum appropriate for the calculated spectra $\{\alpha'_n\}$ and $\{\beta'_n\}$ is written as

$$R_2(\alpha_1^C, \beta_0) = 1 + \sum_{p=1}^{n} (-1)^p (\langle \alpha_p'\beta_{p-1}' \rangle + \langle \alpha_{p-1}'\beta_p' \rangle) + (-1)^n \langle \alpha_n'\beta_n' \rangle. \tag{12}$$

Note that in order to return to the original variables in (12), the entire sum should be multiplied by the exponent $\langle\alpha_1^C\beta_0\gamma_2^C\rangle$, the index of which contains the point taken as the origin of the spectra. The second point of the pair (10) is contained in the expression $\langle\alpha_1^C\beta_0\gamma_2^C\rangle R_2(\alpha_1^C, \beta_0)$. The other pair of points (11) of the section $\gamma = \gamma_2$ is subjected to the same manipulations. Taking the point $(\alpha_0, \beta_1^D, \gamma_2^D)$ as the origin of the spectra, we introduce new variables:

$$\alpha'' = \alpha - \alpha_0; \ \beta'' = \beta - \beta_1^D.$$

On the ellipse $\partial_2(\alpha, \beta) = 0$ the following recurrence relations are obtained:

$$\alpha_n'' = A_{12}(\alpha_0, \beta_1^D)P_n(B) - A_{22}(\alpha_0, \beta_1^D)Q_n(B);$$
$$\beta_n'' = -A_{12}(\alpha_0, \beta_1^D)Q_n(B) + A_{22}(\alpha_0, \beta_1^D)P_n(B);$$

$$A_{12}(\alpha_0, \beta_1^D) = A_1 - 2C\gamma_2^D - 2B\beta_1^D - 2\alpha_0; \tag{13}$$
$$A_{22}(\alpha_0, \beta_1^D) = A_2 - 2D\gamma_2^D - 2B\alpha_0 - 2\beta_1^D.$$

The spectra $\{\alpha''_n\}$, $\{\beta''_n\}$, calculated from (13) allow obtainment of their appropriate closing sum:

$$R_2(\alpha_0, \beta_1^D) = 1 + \sum_{p=1}^{n} (-1)^p (\langle \alpha''_p \beta''_{p-1} \rangle + \langle \alpha''_{p-1} \beta''_p \rangle) + (-1)^n \langle \alpha''_n \beta''_n \rangle . \tag{14}$$

In order to return to the earlier variables, we multiply the entire sum (14) by the exponent with the initial point $\langle \alpha_0 \beta_1^D \gamma_2^D \rangle$. Then the product $\langle \alpha_0 \beta_1^D \gamma_2^D \rangle R_2(\alpha_0, \beta_1^D)$ will contain the second point from (11).

Next, combine the closing sums obtained from the section $\gamma = \gamma_2$ into the total closing sum $R(\gamma_2)$:

$$R(\gamma_2) = \langle \alpha_1^C \beta_0 \gamma_2^C \rangle R_2(\alpha_1^C, \beta_0) + \langle \alpha_0 \beta_1^D \gamma_2^D \rangle R_2(\alpha_0, \beta_1^D).$$

Analogously, the closing sums $R(\gamma_p)$ are calculated for the sections $\gamma = \gamma_p$, $p = 3, 4, \dots, n$.

Designate two spectra determined in (7) in terms of $S(\beta_0)$. For the sections $\beta = \beta_p$, using the algorithm, described above for $\gamma = \gamma_p$, we determine the spectra $S(\beta_p)$. In the same manner, the spectra $S(\alpha_p)$ are determined for the sections $\alpha = \alpha_p$. The spectra $S(\beta_p)$ and $S(\alpha_p)$ contain the values of γ' which are absent in the spectrum $S(\beta_0)$. These new γ make it possible to obtain new sections of the ellipsoid $\gamma = \gamma_{n+m}$, $m = 1, 2, \dots, e$ and write, for these sections, the appropriate closing sums, from which the desired solution is composed

$$U = U_\infty \left(\sum_{p=0}^{n+e} (-1)^p R(\gamma_p) + \dots \right);$$
$$R(\gamma_0) = \langle \alpha_0 \beta_0 \gamma_0 \rangle R_0(\alpha_0, \beta_0); \quad R(\gamma_1) = \langle \alpha_0 \beta_0 \gamma_1 \rangle R_1(\alpha_0, \beta_0). \tag{15}$$

The points in (15) designate as yet unknown terms.

Consider the section $\beta = \alpha$ of the ellipsoid (5):

$$Э(\alpha, \alpha, \gamma) = \gamma^2 + 2(1 + B)\alpha^2 + 2(C + D)\alpha\gamma - 2A\alpha - A\gamma = 0. \tag{16}$$

Let $\nu = \alpha[2(1 + B)]^{1/2}$, then the ellipse (16) may be written in the form

$$\gamma^2 + \nu^2 + 2B'\gamma\nu - A'\nu - A\gamma = 0; \quad A' = \frac{2A}{\sqrt{2(1+B)}}; \tag{17}$$

$$B' = \frac{C + D}{\sqrt{2(1+B)}}. \tag{18}$$

Assuming $C = D = B$ in (18) and considering $B' = \cos[\pi s/(r + 1)]$, we may obtain that $-0.5 < B' < 1$. This means that if all three angles of a melted solid wedge are equal, then they must be larger than $\pi/3$ and smaller than π, which follows from geometric considerations as well.

For the ellipse (17), the spectra $\{\gamma_r\}$ and $\{\nu_r\}$ may be built [2] which form a closed chain of r elements for the quantity B':

$$r = \frac{\pi s}{\arccos B'} - 1, \quad s = 1, 2, \dots, r.$$

Thus, the desired solution becomes closed at the r-th step, if the all calculations above are assumed as the 1st step.

Calculations of the 2nd step are started from determination of three sections $\alpha = \alpha_0^{(2)}$, $\beta = \beta_0^{(2)}$, $\gamma = \gamma_0^{(2)}$ which are determined from the closing points (α_n^B, β_n^B), (α_n^C, γ_n^C), (β_n^D, γ_n^D) of the spectra (6), (7), (8):

$$\alpha_0^{(2)} = A_1 - 2B\beta_n^D - 2C\gamma_n^D; \quad \beta_0^{(2)} = A_2 - 2B\alpha_n^C - 2D\gamma_n^C;$$
$$\gamma_0^{(2)} = A_3 - 2C\alpha_n^B - 2D\beta_n^B.$$

These sections allow calculation of the spectra $S(\alpha_0^{(2)})$, $S(\beta_0^{(2)})$, $S(\gamma_0^{(2)})$ which, also as the spectra (6), (7), (8), give at the first step the onset of calculations of the second step. So, summands of the second step are added to the solution (15). Thus, the desired solution acquires the form:

$$U = U_\infty \sum_{q=1}^{r} \sum_{p=0}^{n+e} (-1)^p R(\gamma_p^{(q)}). \tag{19}$$

Finally, it is worth noting that the presented algorithm allows a solution to be written not only in the case when the all three angles of the melted solid wedge are equal but also when only two angles are equal. This follows from the fact that in (18) only C and D are equivalent.

Example of a Particular Solution. Let the faces of a melted solid wedge be equally tilted about the fusion axis, i.e., $A_1 = A_2 = A_3 = A$. Prescribe two angles with the aperture $\pi/3$ and the third angle $\pi/2$ between its faces. The cosines are between normals to the faces $C = D = -1/2$, $B = 0$. From formula (18) for these quantities $B' = \sqrt{2}/2$ which corresponds to the angle $\pi/4$ and determines the number of calculation steps as $r = 3$. From (10), (12), (14), the initial spectra are calculated:

$$S(\gamma_0) = \{\alpha_0 = 0;\quad \alpha_1 = A;\quad \beta_0 = 0;\quad \beta_1 = A\};$$
$$S(\beta_0) = \{\alpha_0 = 0;\quad \alpha_1 = A;\quad \alpha_2 = 2A;\quad \gamma_0 = 0;\quad \gamma_1 = A;\quad \gamma_2 = 2A\}.$$

Since $C = D$, then the spectrum $S(\alpha_0)$ contains the same values of γ as the spectrum $S(\beta_0)$ has. Correspondingly to the obtained γ, the closing sums are written:

$$R(\gamma_0) = 1 - \langle 0,\ A \rangle - \langle A,\ 0 \rangle + \langle A,\ A \rangle;$$
$$R(\gamma_1) = \langle 0,\ 0,\ A \rangle - \langle 2A,\ 0,\ A \rangle - \langle 0,\ 2A,\ A \rangle + \langle 2A,\ 2A,\ A \rangle;$$
$$R(\gamma_2) = \langle A,\ 0,\ 2A \rangle - \langle 2A,\ 0,\ 2A \rangle - \langle A,\ 3A,\ 2A \rangle +$$
$$+ \langle 2A,\ 3A,\ 2A \rangle + \langle 0,\ A,\ 2A \rangle - \langle 3A,\ A,\ 2A \rangle -$$
$$- \langle 0,\ 2A,\ 2A \rangle + \langle 3A,\ 2A,\ 2A \rangle.$$

The closing point (α_1, β_1) of the section $\gamma_0 = 0$ gives the value of $\gamma_0^{(2)}$ of the initial section of the second calculation step:

$$\gamma_0^{(2)} = A - 2C\alpha_1 - 2D\beta_1 = A + A + A = 3A.$$

For the section $\gamma_0^{(2)} = 3A$, the closing sum is

$$R(\gamma_0^{(2)}) = \langle A,\ A,\ 3A \rangle - \langle 3A,\ A,\ 3A \rangle - \langle A,\ 3A,\ 3A \rangle + \langle 3A,\ 3A,\ 3A \rangle.$$

The closing point of the section $\gamma = \gamma_1$ has the coordinates (2A, 2A, A) which permit one to obtain $\gamma_0^{(3)}$ of the section of the third calculation step:

$$\gamma_0^{(3)} = A - 2C\alpha_{1,1} - 2D\beta_{1,1} - \gamma_1 = A + 2A + 2A - A = 4A.$$

For the section $\gamma_0^{(3)} = 4A$, the closing sum is:

$$R(\gamma_0^{(3)}) = \langle 2A,\ 2A,\ 4A \rangle - \langle 2A,\ 3A,\ 4A \rangle - \langle 3A,\ 2A,\ 4A \rangle + \langle 3A,\ 3A,\ 4A \rangle.$$

Calculation of γ in the spectra $S(\beta_1)$, $S(\beta_2)$, $S(\beta_0^{(2)})$, $S(\beta_0^{(3)})$ yields no new γ values. Therefore, the desired solution is as follows:

$$U = U_\infty (R(\gamma_0) - R(\gamma_1) + R(\gamma_2) - R(\gamma_0^{(2)}) + R(\gamma_0^{(3)})).$$

NOTATION

Σ, surface of the melted solid wedge; (x, y, z), rectangular Cartesian coordinates; v_0, velocity of forward movement of the solid wedge surface relatively material points of this wedge; a, thermal diffusivity; U_∞, temperature at the solid points being at infinite distance from Σ; ξ_1, ξ_2, ξ_3, auxiliary variables; φ_i, ψ_i, θ_i, slopes of the normal to the surface Σ to the z axis; A_1, A_2, A_3, B, C, D, A′, B′, α, β, γ, ν, auxiliary constants; P_n, Q_n, polynomials; m, n, e, p, q, r, s, natural numbers.

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